Engineering Notes

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Simple Method for Prediction of Aircraft Noise Contours

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Introduction

COMMUNITY noise impact is recognized as one of the most serious problems of aircraft operations. The importance of this problem has been highlighted by the increased public concern with environmental issues. To deal with this problem, NASA is engaged in research on technical approaches for noise reduction concepts applicable to new aircraft.

A key tool in the study of noise impact has been noise-contour calculation programs, a number of which have been developed over the last 10 years.[1-3] These programs and their many modifications have been widely used by the government and by industry. However, there are many difficulties in the use of these programs. They require a large computer and a sustained, major effort for use. They are also quite complex and slow, and are generally not desirable for on-line use with piloted simulators. For these reasons, another method that generates noise contours much more rapidly and simply, utilizing minimum computational facilities, is desirable.

In this paper an analytical approach for generating noise contours that overcomes these difficulties is discussed. The method is valid for an arbitrarily complex flight path. For a simple path, the calculations can be accomplished by hand; for a complex path, the calculations are best done on a small hand-held calculator with programming capability, such as the HP-67. The analysis reveals the fundamental nature of the contour and how the various factors that influence its shape and size enter into the analysis. The method is fast, simple, and gives the area, the contour, and its extremities for an arbitrary flight path for both takeoffs and landings.

Theory

The setting for the noise problem is illustrated in Fig. 1 by the projection of a general three-dimensional takeoff trajectory onto the horizontal plane. The trajectory is assumed to consist of straight-line segments as shown. Way-point 1 is the start of takeoff roll located a distance AC from the origin, way-point 2 is liftoff, and the general nth segment is between the nth and $(n+1)$ way points. Each segment is characterized by: a direction angle ψ as indicated, an inclination angle γ measured positive above the horizontal, a segment length s, and a distance R from the aircraft corresponding to the desired EPNdB noise contour and specified thrust level being used on that segment. One may ignore, for the present, the (′) and (″) coordinate systems. From this geometric setting, the most important features of the contour for the takeoff will be deduced. Other situations and conditions will be considered later.

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Noise Contour

The noise contour will be composed of contributions due to each of the various segments, and the various pieces of the contour must be determined and suitably pieced together so as to generate the complete contour. Since the contour for the single segment is a building block for the total contour, it will be expedient to consider first the contour due to a straight-line flight path at an arbitrary inclination angle γ passing through the nth segment as shown in Fig. 2, where the $x'y'$ plane is the horizontal ground plane. The aircraft, when represented by an isotropic noise source, will generate a constant noise-level surface in the shape of a cylinder as shown (or perhaps some other shape if anisotropic). The intersection of this surface with the horizontal plane will give the noise contour on the ground.

To determine the noise contour, it will be necessary to define two orthogonal coordinate systems. The first system, the u, v, w system, as illustrated in Fig. 2, is defined such that the u axis is aligned with the segment, the origin is located at the intersection of the u axis and the horizontal plane, and the v axis lies in the horizontal plane. The second system, the x',y',z' system, is defined with the same origin and such that the x' axis is the projection of the u axis on the horizontal plane, and the y' axis is coincident with the v axis.

With this background, the contour can now be represented as the intersection of two general surfaces, a quadric and a plane, given respectively by:

$$U^T \mathcal{A} U = R^2 \tag{1}$$

$$m^T \mathcal{X} = k \tag{2}$$

The first equation represents the noise-level surface being generated by the aircraft in terms of the first coordinate system, where $\mathcal{A}$ is a matrix, $U=(u\ v\ w)^T$ is a general vector, and R is a constant dependent on the specific noise-level surface. Since this surface is cylindrical with its axis aligned

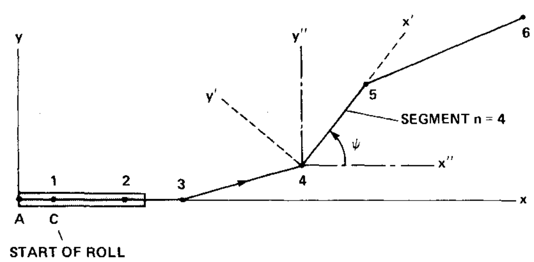

Fig. 1 Plan view of general three-dimensional trajectory.

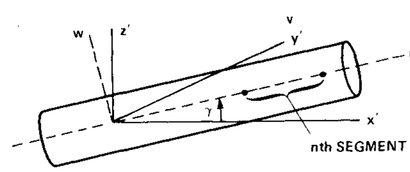

Fig. 2 Three-dimensional geometry and coordinate systems.

with the flight path,

$$\mathfrak{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The second equation represents a plane in terms of the second coordinate system, where $\mathfrak{X} = (x'\ y'\ z')^T$ is a general vector, $m = (m_1\ m_2\ m_3)^T$ is a specific vector with m_i constant, and k is a scalar. Since for the ground plane, $m^T = (0\ 0\ m_3)$ and $k=0$, $\mathfrak{X}$ is orthogonal to m and is of the form

$$\mathfrak{X} = (x'\ y'\ 0)^T \tag{4}$$

To obtain the intersection of Eqs. (1) and (2), Eq. (1) is transformed to the $\mathfrak{X}$ coordinate system by

$$\mathfrak{X} = \Omega U \tag{5}$$

where

$$\Omega = \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix}$$

Equation (1) becomes

$$\mathfrak{X}^T \Omega\ \mathfrak{A}\ \Omega^T \mathfrak{X} = R^2 \tag{6}$$

The intersection is obtained by combining Eqs. (4) and (6). Let $\mathfrak{X}$ be partitioned such that $\mathfrak{X} = (x'\ \ y' \vdots 0)^T \equiv (X'\ \ 0)^T$. Then it can be shown[4] that

$$X'^T A X' = R^2 \tag{7}$$

where

$$A = \begin{bmatrix} \sin^2\gamma & 0 \\ 0 & 1 \end{bmatrix}$$

$$X' = (x'\ y')^T$$

Physically, Eq. (7) represents the noise contour in the horizontal ground plane due to the single straight-line flight path. It is a conic section, and in particular, an ellipse, since the diagonal elements are greater than zero. Note that only two parameters, γ and R, are required to define the contour. Of these, R can be determined from the usual basic noise-source data[1-3] by using it in an inverse manner, that is, by determining the range R for a desired EPNdB level and thrust level.[4] Also note that Eq. (7) applies to each segment, and that each segment contributes only a portion of its conic section to the total contour.

To obtain the total contour, the contributions due to the various segments of the flight path must be properly pieced together. For this purpose, the pieces of the contour are transformed to the x,y coordinate system. With reference to Fig. 1, rotation and translation of the nth segment are accomplished, respectively, by

$$X' = \Lambda_n X'' \tag{8}$$

$$X = X'' + \Delta_n \tag{9}$$

where

$$\Lambda_n = \begin{bmatrix} \cos\psi_n & \sin\psi_n \\ -\sin\psi_n & \cos\psi_n \end{bmatrix}$$

and[4]

$$\Delta_n \equiv \begin{bmatrix} \Delta_{xn} \\ \Delta_{yn} \end{bmatrix} = \sum_{k=1}^{n-1} l_k s_k - l_n (\tan\gamma_n)^{-1} \sum_{k=1}^{n-1} s_k \tan\gamma_k + \begin{bmatrix} \mathrm{AC} \\ 0 \end{bmatrix}$$

where l_n is the direction cosine vector, s_n is the length of the nth segment, and AC is defined in Fig. 1.

Thus, Eq. (7) becomes

$$(X-\Delta_n)^T \Lambda_n^T A_n \Lambda_n (X-\Delta_n) = R_n^2 \tag{10}$$

Physically, this represents the equation of the contour generated by an aircraft flying along any extended nth segment in terms of the x,y coordinate system. Note that this equation applies to each segment but that the parameters for each nth segment are different. Also note that in addition to the previous γ and R, there are now two more quantities Δ and Λ involved and these are functions of the geometry of the flight path as seen from their equations. The contour can be plotted from Eq. (10). For any given choice of x, the value of y is readily obtained, since Eq. (10) is a quadratic in $(y-\Delta_{yn})$. The domain of x for each segment is determined such that x lies closest to that segment as discussed in Ref. 4.

Contour Area

The total area of the contour will be composed of contributions due to each of the various segments of the flight path. The first segment, which is on the ground, can be seen from Fig. 1 to contribute an area

$$A_1 = 2R_1 s_1 + \pi R_1^2/2 \tag{11}$$

The remaining segments are all above the ground and it is necessary to consider the area contributed by the general nth segment. An approximation is made here that the total contour of an arbitrary path is the same as if the trajectory is "straightened out" so that it lies in one vertical plane. Then the area contributed by the nth segment between any two points, denoted by a lower limit (LL) and an upper limit (UL), is, using Eq. (7),

$$A_n(\mathrm{LL},\mathrm{UL}) = 2\int_{\mathrm{LL}}^{\mathrm{UL}} y'(x')\mathrm{d}x' = \Big[x'\sqrt{R_n^2 - (x')^2 \sin^2\gamma_n}$$

$$+ R_n^2 (\sin\gamma_n)^{-1} \sin^{-1}(x' R_n^{-1} \sin\gamma_n) \Big]_{\mathrm{LL}}^{\mathrm{UL}} \tag{12}$$

$$\equiv \Big[F_n \Big]_{\mathrm{LL}}^{\mathrm{UL}} \tag{13}$$

Generally, the lower and upper limits are given, respectively, by the initial coordinate x'_{ni} and the final coordinate x'_{nf} of the flight segment in the x',y',z' coordinate system; however, on the last segment for which the contour exists, the upper limit is given by the end of the contour. It is shown in Ref. 4 that these limits are given in terms of the basic parameters describing the segments by

$$\mathrm{LL} = x'_{ni} \tag{14}$$

$$\mathrm{UL} = \begin{cases} x'_{nf} & \text{if } L_n > x'_{nf} \\ L_n & \text{if } L_n \le x'_{nf} \end{cases} \tag{15}$$

where

$$x'_{ni} = (\tan\gamma_n)^{-1} \sum_{k=1}^{n-1} s_k \tan\gamma_k \tag{16}$$

$$x'_{nf} = x'_{ni} + s_n \tag{17}$$

$$L_n = R_n/\sin\gamma_n \tag{18}$$

If the first inequality in Eq. (15) holds, the end of the contour has not been reached by the end of the segment, while if the second inequality holds, it has. Thus the total area of the contour for N segments is given by

$$A_T = A_1 + \sum_{n=2}^{N} \left[F_n\right]_{x'_{ni}}^{UL} \tag{19}$$

Contour Extremity

The end of the contour is of special interest, and it can be determined from the equations already given. On the last segment, $X'_{max} = (x'_{max} \;\; 0)^T$ and $x'_{max} = L_n$. Then, using the transformations in Eqs. (8) and (9), one obtains

$$X_{max} = X''_{max} + \Delta_n = \Lambda_n^T X'_{max} + \Delta_n \tag{20}$$

Other Considerations

Landing problems have not been considered in the previous analysis. However, it is obvious that such problems can be solved by suitable transformations to an equivalent takeoff problem. The details are discussed in Ref. 4.

Implementation of the preceding equations can be accomplished readily on a small hand-held programmable calculator so as to rapidly determine the quantities of interest, namely, the contour area, the contour, and its extremity for arbitrary flight paths. The repetitive nature of the calculations for successive segments is well-suited to such implementation. A program has been devised for the Hewlett-Packard HP-67, the details of which are given in Ref. 4. The inputs are the parameters describing the successive segments as already discussed as well as parameters describing the noise. The outputs are the contour area, the actual contour in terms of the y coordinates corresponding to any x coordinate, and the coordinates of the extremity of the contour. The program is valid for both takeoff and landing trajectories. Since the equations are so simple, the program executes in seconds. For this reason, the program, when suitably implemented, would be ideally suited to on-line piloted simulation.

Another consideration concerns the accuracy of the closed-form solution. This accuracy has been investigated in Ref. 4 by comparing the results from the closed-form solution with that from the large computer contour program for identical conditions. Many examples for takeoff and landing trajectories of a conventional jet transport aircraft were used, too numerous to detail here. A typical comparison is shown in Fig. 3 by an example two-segment takeoff with thrust cutback in which ground attenuation and shielding have been omitted. Two sets of data are shown. First, the contour represents the results obtained from the program of Refs. 1 and 2 using the IBM-360 computer. The program is about 100,000 bytes in size, and a substantial amount of time and effort was required to obtain these results. Second, the crosses shown represent the results from the closed-form solution using the hand-held HP-67 calculator. Comparison shows excellent agreement. As

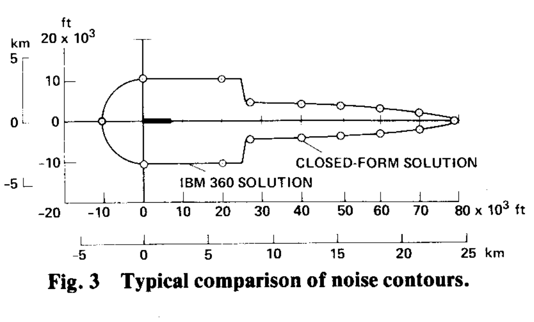

Fig. 3 Typical comparison of noise contours.

Table 1 Comparison of closed-form solution and computer program

	Area, km²	
Example	Closed-form solution	IBM-360 program
1	154.93	154.9
2	154.93	155.4
3	100.49	99.5
4	17.59	17.4
5	24.22	24.1

for the contour areas, the results for several examples[4] are illustrated in Table 1 where it can be seen that the accuracy is within 1%.

Concluding Remarks

The simplicity of the noise analysis presented herein enables one to obtain the noise contour, its area, and its extremities for an arbitrarily complex flight path for both takeoffs and landings. The method is simple and fast, and results can be obtained either by manual calculation or by means of a small programmable calculator. The analysis reveals the fundamental nature of the contours and how the various factors that influence its size and shape enter into the analysis.

It should be noted that the effects of ground attenuation and shielding have been omitted from the analysis. Generally, their effects are important only on the initial portion of flight, and are highly dependent upon aircraft configuration. Preliminary analysis shows that such effects could be included, and further work in this direction might be warranted.

It is also worth emphasizing that the single-event contour discussed herein is the obvious choice for purposes of minimizing noise impact. The impact of multiple flights of the same type are handled by an obvious extension of the single-event results.

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Use of Similitude in Analyzing Aircraft Windshield Anti-Icing Performance

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Nomenclature

f = function in Eq. (1)
P_A = ambient pressure, lb/ft²
P_B = bleed airflow pressure, lb/ft²
T_A = ambient temperature, °R

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